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STOCHASTIC MODELING ON LIKERT'S SCALING MEASURES

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ABSTRACT

In this paper, the focus is on theoretical development of quantification models for Likert's scaling measures to explore the parameters of the population using the stochastic processes. The experiments of two-way spreadsheet quantification were considered to get the stochastic models. Joint discrete probability distributions are derived with the results of experiments. Mathematical relations for statistical measures were derived to the developed model. Numerical illustrations are provided for better understanding of the model at the level of nonprofessional. This study is useful for measuring the research tool score which is in ordinal scales. Derivations of assessment devices, running of inferential study procedures, formulation of optimal decision designing, etc related to scaling measures with Likert's or Symantec methods can be dealt with this study.

KEYWORDS: Stochastic Modeling, Likert's Scaling Measures, Spreadsheet Quantification, Optimal Decision Designing

INTRODUCTION

Technology oriented quantification become the order of the day because of its importance in qualitative judgment methods. Various scaling measures have been used in the assessment and understanding of numerical visibility. Statistical Parameters will measure the population characteristics on numerical lines. They provide the indicators on assessment and evaluation processes. It is a landmark achievement that the qualitative traits have been measured in quantity formats in the contemporary research studies, irrespective of its domain area. Exploring the devices of investigation is the pivotal component for research inferences. They become indispensable in rational understanding of the problem. Therefore, the model may be considered as an anatomy of information structures and data patterns. The computing methods have to be monitored carefully by suitable theoretical models, as the later are the essential driving devices for former. Measures of quantified values have to be equipped with relevant scaling models. Extraction of hidden intelligence of the data is the ultimate purpose of data modeling with scaling measures.

The job satisfactions, performances of employees, time management, etc were studied the theoretical framework[1]. Properties of mixture in stochastic processes and statistics are utilized in statistical down scaling [2]. The investment scale models have been used to study the reliability and validity of psychometric parameters [3]. A framework on multiple spatial and temporal scales was applied in climate studies [4]. Spreadsheet based business models were developed to study the efficiency of business [5]. Financial decisions, health/safety, recreational, ethical, and social decisions were studied with the psychometric scales [6]. A measurement model for assessing enterprise system's success from multiple perspectives measures namely information quality, system quality, individual impact, and organizational impact is validated[7]. Modeling of business concerns with spreadsheet without use of mathematics and statistics was proposed [8]. An additive scale model for the Analytic Hierarchy Process (AHP) was presented by using a linear preference comparison by relating mathematical denotation, axiom, transitivity and numerical analysis [9]. A model for systematic knowledge translation was presented with descriptive summary measures [10]. Surrogate Decision Making (SDM)

Self-Efficacy Scale was used for an assessment of instrument to measure the interventions [11]. *M*ulti criteria decision-making procedural scales were proposed for handling the complex classification of software represented in different data formats [12].

Observing the literature, it is evident that much emphasis was given on psychometric measurement of scaling in the contexts of empirical case related studies only. There is little evidence on development of mathematical modeling of measures like Likert's scale. In this work, a spreadsheet approach of rows and columns is considered for quantifying the item wise score and overall scores.

Here, the rows represent the items for quantification and the columns representing the score on the Likert's / Symantec scale such that each item is measured on the mentioned scale. The prime objective of this work is to identify the Bi-variate stochastic processes for exploring the quantity of the study to provide models of scaling measures. Obtaining different descriptive statistics. Comparative studies of two or more quantification activities can also be done with this study. Numerical data sets were generated on simulation methods to understand the model behaviour with a reach of a layman.

2. STOCHASTIC MODELS

Let there be 'm' number of listed study items in the research tool, each is measured on 'n' points scale. Score for each item is obtained in between 1 to n as integers, the total score on all 'm' items can be obtained by summing of each score. The joint probability function may be obtained through relative frequency distribution when we have the bivariate frequency distribution. The list and number of study items are according to the research tool. Opting the score point to the specific study item depends on the assessment of the item by the respondent.

This score is varying from one respondent to other respondent. Usually the respondent has to select only one score point among 'j' availabilities defined with 'n' points scale. The selection of j^{th} score point eliminates (excludes) the remaining (n-1) score points. While developing the study tool, there are two alternative approaches for considering the study items in the list; (i) all the study items are equally weighted (ii) several study items have different weights to be included in computation of response score. While computing the study score of each response tool, there are two alternative ways; (a) computation in view of score point of the scale (b) computation in view of listed study item. In the former view, row wise scores as per the selected position of 'j' is obtained for each item; whereas in the later view, column wise summation of score points and sum up of all the item's scores. With the above stipulations, we have developed four models on the assumption of each study item in the list is having independent probability distribution. The purpose of all the models is to derive the statistical measures on the score of response.

(1). Selection of *listed item with equal weight*, computation of tool *score in view of scale point*

(2). Selection of *listed item with unequal weight*, computation of tool *score in view of scale point*

(3). Selection of *listed item with equal weight*, computation of tool *score in view of listed item*

(4). Selection of *listed item with unequal weight*, computation of tool *score in view of listed item*

2.1. Stochastic Models with Probability Distributions of Individual Listed Item

In this section, the above set of models is developed with the following assumptions. Let P_{ij} be the probability of opting j^{th} score point for i^{th} study item and it assumes the values as $P_{ij} = 1$; when j^{th} score point is being opted for the i^{th}

study item; and P_{ij} = 0; when j^{th} score point is not being opted for the i^{th} study item; for j=1,2,…..,n; i=1,2,…..m. Let the joint probability distribution for measuring the study scores be denoted as below.

		Score Point (j)						P(i)	k	w_{i1}	w_{i2}	$P(i)_1$	$P(i)_2$
		1	2	…..	J	….	n						
Study item (i)	1	P_{11}	P_{12}	…..	P_{1j}	…..	P_{1n}	$P_{1.}$=1	k_1	w_{11}	w_{12}	$q_{11.}$	$q_{12.}$
	2	P_{21}	P_{22}	…..	P_{2j}	…..	P_{2n}	$P_{2.}$ =1	k_2	w_{21}	w_{22}	$q_{21.}$	$q_{22.}$
	:	:	:	…..	:	…..	:	:	:	:	:	:	:
	i	P_{i1}	P_{i2}	…..	P_{ij}	…..	P_{in}	$P_{i\cdot}$ =1	k_i	w_{i1}	w_{i2}	$q_{i1.}$	$q_{i2.}$
	:	:	:	…..	:	…..	:	:	:	:	:	:	:
	m	P_{m1}	P_{m2}	…..	P_{mj}	…..	P_{mn}	$P_{m\cdot}$=1	k_m	w_{m1}	w_{m2}	$q_{m1.}$	$q_{m2.}$
$P_{(j)1}$		$q_{.11}$	$q_{\cdot 21}$	…..	$q_{\cdot j1}$	…..	$q_{\cdot n1}$	1				1	1
$P_{(j)2}$		$q_{.12}$	$q_{\cdot 22}$	…..	$q_{\cdot j2}$	…..	$q_{\cdot n2}$	1					

2.1.1: Model for Listed Item with Equal Weight, Scoring in View of Listed Study Item

In this model the score is calculated in view of study item with equal weight to each listed item.

Let w_i be the weight of i^{th} study item for quantification of the score Let $q_{i1.} = \frac{w_{i1}}{\sum_{i=1}^{m} w_{i1}}$ be the marginal probability of i^{th} listed study item. Here, w_{i1} is constant. If w_{i1}=c; then $q_{i1.} = \frac{c}{\sum_{i=1}^{m} c} = \frac{c}{m.c} = \frac{1}{m}$; $k_i = \sum_{j=1}^{n} y_j P_{ij}$; is the score component of i^{th} listed study item; and y_j is the scaled score point at j^{th} ordinate; For the given joint distribution, $\sum_{j=1}^{n} P_{ij} = P_{i.} = 1$ for every i=1,2,......m. which implies $\sum_{i=1}^{m} P_{i.} = \sum_{i=1}^{m} 1 = m$,

The statistical measures of the above models are

1. The average score of a study tool is $E(S_{w_1}) = \mu_{01}^{'}(S_{w_1}) = \frac{1}{m}\sum_{i=1}^{m}(\sum_{j=1}^{n} y_j P_{ij})$

2. The variance of the study score is

$$V(S_{w_1}) = \mu_{02}(S_{w_1}) = \frac{1}{m}\sum_{i=1}^{m}(\sum_{j=1}^{n} y_j P_{ij})^2(1-\frac{1}{m}) - \frac{2}{m^2}\sum_{i\neq k=1}^{m}(\sum_{j=1}^{n} y_i P_{ij})(\sum_{j=1}^{n} y_k P_{kj}) \qquad 2.1.1.2$$

3. The third central Moment of study score of a tool is

$$\mu_{03}(S_{w_1}) = \frac{1}{m}\sum_{i=1}^{m}(\sum_{j=1}^{n} y_j P_{ij})^3 - \frac{1}{m^2}\sum_{i=1}^{m}\sum_{i=1}^{m}(\sum_{j=1}^{n} y_j P_{ij})^3(-3+\frac{2}{m}) + \sum_{i=1}^{m}\sum_{i\neq k=1}^{m}(\sum_{j=1}^{n} y_j P_{ij})^2(\sum_{j=1}^{n} y_j P_{kj}) \qquad 2.1.1.3$$

4. The fourth central moment of score of study tool is

$$\mu_{04}(S_{w_1}) = \frac{1}{m}\sum_{i=1}^{m}(\sum_{j=1}^{n} y_j P_{ij})^4 - \frac{1}{m^2}\sum_{i=1}^{m}\sum_{i=1}^{m}(\sum_{j=1}^{n} y_j P_{ij})^4(-4+6-\frac{3}{m^2})$$
$$+\frac{6}{m^2}\sum_{i=1}^{m}\sum_{i\neq k=1}^{m}(\sum_{j=1}^{n} y_j P_{ij})^3(\sum_{j=1}^{n} y_j P_{kj})(1-\frac{1}{m^2}) - 3\sum_{i\neq k=1}^{m}\sum_{i\neq k=1}^{m}(\sum_{j=1}^{n} y_j P_{ij})^2(\sum_{j=1}^{n} y_j P_{kj})^2 \quad 2.1.1.4$$

2.1.2: Model for Listed Item with Unequal Weight, Scoring in View of Listed Study Item

In this model the study score is calculated in view of listed study item having different weights of they are being involved in the scoring of the research tool. The assumptions of this model are based on usual notion of the model 2.1.1 Let w_{i2} be the weight of i[th] listed item being involved for score quantification. Let $q_{i2.} = \frac{w_{i2}}{\sum_{i=1}^{m} w_{i2}}$ be the marginal probability of i[th] study item.

Here, w_{i2} is varying and it may be allocated with many considerations. Usually, it is assumed that the study items are arranged in the increased order of priority, such that w_{12}=m, w_{22}=m-1. w_{32}=m-2, ….. w_{i2}=m-(i-1),…..w_{m2}=1; $k_i = \sum_{j=1}^{n} y_j P_{ij}$; is the score component of i[th] study item; and y_j is the score point at j[th] ordinate.

The statistical measures of the model are

1. The average score of the study tool is

$$E(S_{w_2}) = \mu_{01}^{'}(S_{w_2}) = \frac{1}{M}\sum_{i=1}^{m} w_{i2}(\sum_{j=1}^{n} y_j P_{ij}); M = \sum_{i=1}^{m} w_{i2} \quad 2.1.2.1$$

2. The variance of the study score is

$$\mu_{02}(S_{w_2}) = \frac{1}{M}\sum_{i=1}^{m} w_{i2}(\sum_{j=1}^{n} y_j P_{ij})^2(1-\frac{1}{M}) - \frac{2}{M^2}\sum_{i\neq k=1}^{m} w_{i2}w_{k2}(\sum_{j=1}^{n} y_i P_{ij})(\sum_{j=1}^{n} y_k P_{kj}) \quad 2.1.2.2$$

3. The third central Moment of score of the study tool is

$$\mu_{03}(S_{w_2}) = \frac{1}{M}\sum_{i=1}^{m} w_{i2}(\sum_{j=1}^{n} y_j P_{ij})^3 - \frac{1}{M^2}\sum_{i=1}^{m}\sum_{i=1}^{m}(w_{i2})^2(\sum_{j=1}^{n} y_j P_{ij})^3(-3+\frac{2}{M}) + \sum_{i=1}^{m}\sum_{i\neq k=1}^{m} w_{i2}w_{k2}(\sum_{j=1}^{n} y_j P_{ij})^2(\sum_{j=1}^{n} y_j P_{kj}) \quad 2.1.2.3$$

4. The fourth central moment of study score of the tool is

$$\mu_{04}(S_{w_2}) = \frac{1}{M}\sum_{i=1}^{m} w_{i2}(\sum_{j=1}^{n} y_j P_{ij})^4 - \frac{1}{M^2}\sum_{i=1}^{m}\sum_{i=1}^{m}(w_{i2})^2(\sum_{j=1}^{n} y_j P_{ij})^4(-4+6-\frac{3}{M^2})$$
$$+\frac{6}{M^2}\sum_{i=1}^{m}\sum_{i\neq k=1}^{m} w_{i2}w_{k2}(\sum_{j=1}^{n} y_j P_{ij})^3(\sum_{j=1}^{n} y_j P_{kj})(1-\frac{1}{M^2}) - 3\sum_{i\neq k=1}^{m}\sum_{i\neq k=1}^{m}(w_{k2})^2(\sum_{j=1}^{n} y_j P_{ij})^2(\sum_{j=1}^{n} y_j P_{kj})^2 \quad 2.1.2.4$$

2.1.3: Model for Listed Item with Equal Weight, Scoring in View of Scaled Score Point

In this model, the quantification of study tool’s score is calculated in view of listed study item, where each listed items have fixed weight of it is being considered for study.

Let $q_{.j1} = \frac{\sum_{i=1}^{m} P_{ij} w_{i1}}{\sum_{i=1}^{m} w_{i1}}; w_{i1} = c$ be the marginal probability of jth scaled score point.

The simplified value of it is $q_{.j1} = \frac{\sum_{i=1}^{m} P_{ij} c}{\sum_{i=1}^{m} c} = \frac{c\sum_{i=1}^{m} P_{ij}}{m.c} = \frac{\sum_{i=1}^{m} P_{ij}}{m}$

As P_{ij} is either 1 or 0, which implies that $\sum_{i=1}^{m} P_{ij} = m_j$; for j=1,2,…..n. It implies that $q_{.j1} = \frac{m_j}{m}$

The statistical measures of the model are

1. The average study score of the research tool is $\mu'_{10}(S_{w_1}) = \frac{1}{m}\sum_{j=1}^{n} jm_j$ 2.1.3.1

2. The variance of the study score of a tool is 2.1.3.2

3. The third central Moment is $\mu_{30}(S_{w_1}) = \frac{1}{m}\sum_{j=1}^{n} j^3 m_j - \frac{1}{m^2}\sum_{j\neq k=1}^{n}(m_j)^2\left[\sum_{j=1}^{n} j^3(3-\frac{2m_j}{m}) - \frac{2}{m}\sum_{j\neq k=1}^{n} jk(m_k)\right]$ 2.1.3.3

4. The fourth central moment is

$$\mu_{40}(S_{w_1}) = \frac{1}{m}\sum_{j=1}^{n} j^4(m_j) - \frac{1}{m^2}\sum_{j=1}^{n}\sum_{j=1}^{n} j^4(m_j)^2\left(-4+\frac{6m_j}{m}-\frac{3(m_j)^2}{m^2}\right)$$
$$+\frac{3}{m^4}\left(2\sum_{j=1}^{n}\sum_{j\neq k=1}^{n} j^3.k(m_j)^2(m_k)(m-m_j) - \sum_{j\neq k=1}^{n}\sum_{j\neq k=1}^{n} j^2k^2(m_j)^2(m_k)^2\right)$$
2.1.3.4

2.1.4: Model for Listed Item with Unequal Weight, Scoring in View of Scaled Score Point

In this model the score of the study tool is calculated in view of listed study item where the items have unequal weights of each item. Let $q_{.j2} = \frac{\sum_{i=1}^{m} P_{ij} w_{i2}}{\sum_{i=1}^{m} w_{i2}}$; ($w_{i2}$ is not a constant), be the marginal probability of jth scale point. The simplified value of it is $q_{.j2} = \frac{\sum_{i=1}^{m} P_{ij} w_{i2}}{M}; M = \sum_{i=1}^{m} w_{i2}$

The statistical measures of the model are

1. The average study score of the tool is $\mu'_{10}(S_{w2}) = \frac{1}{M}\sum_{j=1}^{n} jw_{i2}P_{ij}$ 2.1.4.1

2. The variance of the score of the study tool is

$$\mu_{20}(S_{w2})=\frac{1}{M^2}\left(\sum_{j=1}^{n} j^2 w_{i2}P_{ij}(M-w_{i2}P_{ij})-\sum_{j\neq k=1}^{n} jkw_{i2}w_{k2}P_{ij}P_{kj}\right) \quad 2.1.4.2$$

3. The third central Moment of the score of the study tool is

$$\mu_{30}(S_{w2})=\frac{1}{M}\sum_{j=1}^{n} j^3 w_{i2}P_{ij}-\frac{1}{M^2}\sum_{j\neq k=1}^{n}(w_{i2}P_{ij})^2\left[\sum_{j=1}^{n} j^3(3-\frac{2w_{i2}P_{ij}}{M})-\frac{2}{M}\sum_{j\neq k=1}^{n} jk(w_{k2}P_{kj})\right] \quad 2.1.4.3$$

4. The fourth central moment of the score of the study tool is

$$\mu_{40}(S_{w2})=\frac{1}{M}\sum_{j=1}^{n} j^4(w_{i2}P_{ij})-\frac{1}{M^2}\sum_{j=1}^{n}\sum_{j=1}^{n} j^4(w_{i2}P_{ij})^2\left(-4+\frac{6w_{i2}P_{ij}}{M}-\frac{3(w_{i2}P_{ij})^2}{M^2}\right)$$
$$+\frac{3}{M^4}\left(2\sum_{j=1}^{n}\sum_{j\neq k=1}^{n} j^3.k(w_{i2}P_{ij})^2(w_{k2}P_{kj})(M-w_{i2}P_{ij})-\sum_{j\neq k=1}^{n}\sum_{j\neq k=1}^{n} j^2k^2(w_{i2}P_{ij})^2(w_{k2}P_{kj})^2\right) \quad 2.1.4.4$$

2.2. Stochastic Models with Joint Probability Distribution of Listed Item and Score Points

The models 2.1.1 to 2.1.4 are the different special cases for computing the score of response research tool using a probability distribution. In these models, the problem deals with 'm' individual probability distributions separately and clubbing them to get the overall score. This section deals with two models 2.2.1 & 2.2.2 are formulated based on the joint probability distribution of 'm' listed items and 'n' score points for each item combined.

2.2.1 Model on Listed Items with Equal Weight and the Score in View of Listed Items

The following joint probability distribution is considered for the development of models 2.2.1 & 2.2.2.

		Score Point (j)						P(i) (1)	k	wi
		1	2	…..	j	….	n			
Study item (i)	1	$q_{11(1)}$	$q_{12(1)}$	…..	$q_{1j(1)}$	…..	$q_{1n(1)}$	$q_{1.(1)}$	k_1	w
	2	$q_{21(1)}$	$q_{22(1)}$	…..	$q_{2j(1)}$	…..	$q_{2n(1)}$	$q_{2.(1)}$	k_2	w
	:	:	:	…..	:	…..	:	:	:	:
	i	$q_{i1(1)}$	$q_{i2(1)}$	…..	$q_{ij(1)}$	…..	$q_{in(1)}$	$q_{i\cdot(1)}$	k_i	w
	:	:	:	…..	:	…..	:	:	:	:
	m	$q_{m1(1)}$	$q_{m2(1)}$	…..	$q_{mj(1)}$	…..	$q_{mn(1)}$	$q_{m\cdot(1)}$	k_m	w
P(j) (1)		$q_{.1(1)}$	$q_{\cdot 2(1)}$	…..	$q_{.j(1)}$	…..	$q_{.n(1)}$	1		

Let q_{ij} be the probability of opting j^{th} score point for i^{th} study item, defined as

$q_{ij(1)}=\frac{P_{ij}w_i}{\sum_{i=1}^{m}w_i}=\frac{P_{ij}w}{\sum_{i=1}^{m}w}=\frac{P_{ij}}{m}$ where w_i is constant i.e. w_i=c; P_{ij} = 1; when j^{th} score point is being opted for the i^{th} study item; and P_{ij} = 0; when j^{th} score point is not being opted for the i^{th} study item; for j=1,2,…..,n; i=1,2,…..m

$q_{i.(1)}=\sum_{j=1}^{n}q_{ij(1)}$ is the marginal probability of i^{th} study item and $q_{.j(1)}=\sum_{i=1}^{m}q_{ij(1)}$ is the marginal probability of j^{th} score point. For the given joint distribution, $\sum_{j=1}^{n}q_{.j(1)}=\sum_{i=1}^{m}q_{i.(1)}=1$. Let $k_i=\sum_{j=1}^{n}y_jP_{ij}$ be the score component of i^{th} study item. y_j is the score point at j^{th} ordinate.

The statistical measures of the model are

1. The average score of a study tool is $E(S_{w_1}) = \mu'_{10}(S_{w_1}) = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(1)}\right)$ 2.2.1.1

2. The variance of the study score is

$$V(S_{w_1}) = \mu_{20}(S_{w_1}) = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)^2\left(\sum_{j=1}^{n} q_{ij(1)}\right) - \left(\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(1)}\right)\right)^2 \quad 2.2.1.2$$

3. The third central Moment of study score of a tool is

$$\mu_{30}(S_{w_1})) = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)^3\left(\sum_{j=1}^{n} q_{ij(1)}\right) - 3\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)^2\left(\sum_{j=1}^{n} q_{ij(1)}\right)\right]\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(1)}\right)\right] + 2\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(1)}\right)\right]^3 \quad 2.2.1.3$$

4. The fourth central moment of score of study tool is

$$\mu_{40}(S_{w_1})) = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)^4\left(\sum_{j=1}^{n} q_{ij(1)}\right) - 4\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)^3\left(\sum_{j=1}^{n} q_{ij(1)}\right)\right]\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(1)}\right)\right] + 6\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)^2\left(\sum_{j=1}^{n} q_{ij(1)}\right)\right]\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(1)}\right)\right]^2 - 3\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(1)}\right)\right]^4 \quad 2.2.1.4$$

2.2.2 Model on Listed Items with Equal Weight and the Score in View of Scale Points

In this model the study score is calculated in view of the item of study tool with equal weights. The joint probability distribution for measuring the study scores including the assumptions with Joint and Marginal probabilities are as in the model 2.2.1.

The statistical measures of the model are

1. The average score of a study tool is $E(S_{w_1}) = \mu'_{01}(S_{w_1}) = \sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(1)}$ 2.2.2.1

2. The variance of the study score is

$$V(S_{w_1}) = \mu_{02}(S_{w_1}) = \sum_{i=1}^{m}\sum_{j=1}^{n} j^2 q_{ij(1)} - \left(\sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(1)}\right)^2 \quad 2.2.2.2$$

3. The third central Moment of study score of a tool is

$$\mu_{03}(S_{w_1})) = \sum_{i=1}^{m}\sum_{j=1}^{n} j^3 q_{ij(1)} - 3\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j^2 q_{ij(1)}\right)\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(1)}\right) + 2\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(1)}\right)^3 \quad 2.2.2.3$$

4. The fourth central moment of score of study tool is

$$\mu_{04}(S_{w_1}) = \sum_{i=1}^{m}\sum_{j=1}^{n} j^4 q_{ij(1)} - 4\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j^3 q_{ij(1)}\right)\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(1)}\right) + 6\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j^2 q_{ij(1)}\right)\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(1)}\right)^2 - 3\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(1)}\right)^4 \quad 2.2.2.4$$

2.2.3 Model on Listed Items with Unequal Weight and the Score in View of Listed Items

The following joint probability distribution is considered for the development of models 2.2.3 & 2.2.4

		Score Point (j)						**P(i) (2)**	**k**	**wi**
		1	**2**	**…..**	**j**	**….**	**n**			
Study item (i)	1	$q_{11(2)}$	$q_{12(2)}$	…..	$q_{1j(2)}$	…..	$q_{1n(2)}$	$q_{1.(2)}$	k_1	w_1
	2	$q_{21(2)}$	$q_{22(2)}$	…..	$q_{2j(2)}$	…..	$q_{2n(2)}$	$q_{2.(2)}$	k_2	w_2
	:	:	:	…..	:	…..	:	:	:	:
	i	$q_{i1(2)}$	$q_{i2(2)}$	…..	$q_{ij(2)}$	…..	$q_{in(2)}$	$q_{i\cdot(2)}$	k_i	w_i
	:	:	:	…..	:	…..	:	:	:	:
	m	$q_{m1(2)}$	$q_{m2(2)}$	…..	$q_{mj(2)}$	…..	$q_{mn(2)}$	$q_{m\cdot(2)}$	k_m	w_m
$P(j)_{(2)}$		$q_{.1(2)}$	$q_{\cdot 2(2)}$	…..	$q_{.j(2)}$	…..	$q_{.n(2)}$	1		

Let q_{ij} be the probability of opting j^{th} score point for i^{th} study item, defined as

$$q_{ij(2)} = \frac{P_{ij} w_i}{\sum_{i=1}^{m} w_i}$$ where w_i is a variable i.e. $w_i \neq c$; $P_{ij} = 1$; when j^{th} score point is being opted for the i^{th} study item;

and $P_{ij} = 0$; when j^{th} score point is not being opted for the i^{th} study item; for j=1,2,…..,n; i=1,2,…..m

$q_{i.(2)} = \sum_{j=1}^{n} q_{ij(2)}$ is the marginal probability of i^{th} study item and $q_{.j(2)} = \sum_{i=1}^{m} q_{ij(2)}$ is the marginal probability of j^{th} score point. For the given joint distribution, $\sum_{j=1}^{n} q_{.j(2)} = \sum_{i=1}^{m} q_{i.(2)} = 1$. Let $k_i = \sum_{j=1}^{n} y_j P_{ij}$ be the score component of i^{th} study item. y_j is the score point at j^{th} ordinate.

The statistical measures of the model are

1. The average score of a study tool is $E(S_{w_2}) = \mu_{10}'(S_{w_2}) = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(2)}\right)$ 2.2.3.1

2. The variance of the study score is

$$V(S_{w_2}) = \mu_{20}(S_{w_2}) = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)^2\left(\sum_{j=1}^{n} q_{ij(2)}\right) - \left(\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(2)}\right)\right)^2 \quad 2.2.3.2$$

3. The third central Moment of study score of a tool is

$$\mu_{30}(S_{w_2})) = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)^3 \left(\sum_{j=1}^{n} q_{ij(2)}\right) - 3\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)^2 \left(\sum_{j=1}^{n} q_{ij(2)}\right)\right]\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(2)}\right)\right] + 2\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(2)}\right)\right]^3 \quad 2.2.3.3$$

4. The fourth central moment of score of study tool is

$$\mu_{40}(S_{w_2})) = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)^4 \left(\sum_{j=1}^{n} q_{ij(2)}\right) - 4\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)^3 \left(\sum_{j=1}^{n} q_{ij(2)}\right)\right]\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(2)}\right)\right] + 6\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)^2 \left(\sum_{j=1}^{n} q_{ij(2)}\right)\right]\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(2)}\right)\right]^2 - 3\left[\sum_{i=1}^{m}\left(\sum_{j=1}^{n} y_j P_{ij}\right)\left(\sum_{j=1}^{n} q_{ij(2)}\right)\right]^4 \quad 2.2.3.4$$

2.2.4 Model on Listed Items with Unequal Weight and the Score in View of Scale Points

In this model the study score is calculated in view of the item of study tool with equal weights. The joint probability distribution for measuring the study scores including the assumptions with Joint and Marginal probabilities as in the model 2.2.3.

The statistical measures of the model are

1. The average score of a study tool is $E(S_{w_2}) = \mu_{01}^{'}(S_{w_2}) = \sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(2)}$ 2.2.4.1

2. The variance of the study score is

$$V(S_{w_2}) = \mu_{02}(S_{w_2}) = \sum_{i=1}^{m}\sum_{j=1}^{n} j^2 q_{ij(2)} - \left(\sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(2)}\right)^2 \quad 2.2.4.2$$

3. The third central Moment of study score of a tool is

$$\mu_{03}(S_{w_2}) = \sum_{i=1}^{m}\sum_{j=1}^{n} j^3 q_{ij(2)} - 3\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j^2 q_{ij(2)}\right)\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(2)}\right) + 2\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(2)}\right)^3 \quad 2.2.4.3$$

4. The fourth central moment of score of study tool is

$$\mu_{04}(S_{w_2}) = \sum_{i=1}^{m}\sum_{j=1}^{n} j^4 q_{ij(2)} - 4\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j^3 q_{ij(2)}\right)\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(2)}\right) + 6\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j^2 q_{ij(2)}\right)\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(2)}\right)^2 - 3\left(\sum_{i=1}^{m}\sum_{j=1}^{n} j q_{ij(2)}\right)^4 \quad 2.2.4.4$$

3. NUMERICAL ILLUSTRATION AND ANALYSIS

In order to understand the above mentioned stochastic models with a reach of common researcher, a study tool for quantification with 19 listed items each is measured on 8 point scale was considered for a model study tool.

Table 3.1: The Study Scores with Fixed and Varying Priorities of Selecting Listed Study Items for Models 2.1.1 to 2.1.4

Study Item (i)[1]	Score Point Scale (j) [2]								P_{i0} [3]	K [4]	Equal Weights		Unequal Weight	
	1	2	3	4	5	6	7	8			Wt1 [5]	$q_{i0}(1)$ [6]	wt-2 [7]	$q_{i0}(2)$ [8]
1	0	0	1	0	0	0	0	0	1	5	1	0.053	19	0.1
2	0	0	0	1	0	0	0	0	1	4	1	0.053	18	0.095
3	0	0	0	0	0	1	0	0	1	6	1	0.053	17	0.089
4	0	0	0	1	0	0	0	0	1	4	1	0.053	16	0.084
5	0	0	0	0	0	0	0	1	1	8	1	0.053	15	0.079
6	0	1	0	0	0	0	0	0	1	2	1	0.053	14	0.074
7	1	0	0	0	0	0	0	0	1	1	1	0.053	13	0.068
8	0	0	0	0	0	0	1	0	1	7	1	0.053	12	0.063
9	0	0	0	1	0	0	0	0	1	4	1	0.053	11	0.058
10	0	0	0	0	0	1	0	0	1	6	1	0.053	10	0.053
11	0	0	1	0	0	0	0	0	1	3	1	0.053	9	0.047
12	0	0	0	0	1	0	0	0	1	5	1	0.053	8	0.042
13	0	1	0	0	0	0	0	0	1	2	1	0.053	7	0.037
14	0	0	0	0	0	0	1	0	1	7	1	0.053	6	0.032
15	1	0	0	0	0	0	0	0	1	1	1	0.053	5	0.026
16	0	0	0	1	0	0	0	0	1	4	1	0.053	4	0.021
17	0	0	0	0	0	1	0	0	1	6	1	0.053	3	0.016
18	0	0	0	0	1	0	0	0	1	5	1	0.053	2	0.011
19	0	0	0	1	0	0	0	0	1	4	1	0.053	1	0.005
m_j [10]	2	4	3	20	15	18	14	8		84	19	1	190	1
$q_{0j}(1)$ [11]	0.105	0.105	0.053	0.26	0.16	0.158	0.105	0.053		1				
$q_{0j}(2)$ [12]	0.095	0.111	0.047	0.26	0.15	0.158	0.095	0.079		1				

Table 3.2: Various Descriptive Statistics with Fixed and Varying Weights of Study Items for Models 2.1.1. to 2.1.4

Statistical Measure Measure	Model-2.1.1 (Fixed Weights)	Model 2.1.2 (Varying Weights)	Model-2.1.3 (Fixed Weights)	Model-2.1.4 (Varying Weights)
Myu-1' (Average)	4.4211	4.5158	4.4211	4.5158
Myu-2'	23.3684	24.3684	23.3684	24.3684
Myu-3'	136.1053	145.2000	136.1053	145.2000
Myu-4'	845.0526	923.9263	845.0526	923.9263
Mean	4.4211	4.5158	4.4211	4.5158
Myu-2 (Variance)	3.8227	3.9761	3.8227	3.9761
S.D.	1.9552	1.9940	1.9552	1.9940
C.V.	0.4422	0.4416	0.4422	0.4416
Myu-3	-1.0086	-0.7528	-1.0086	-0.7528
Beta-1	0.0182	0.0090	0.0182	0.0090
Gamma-1	0.1349	0.0950	0.1349	0.0950
Myu-4	32.5484	35.1883	32.5484	35.1883
Beta-2	2.2273	2.2258	2.2273	2.2258
Gamma-2	-0.7727	-0.7742	-0.7727	-0.7742

Table-3.1 deals with the numerical illustration of a spontaneous response to the listed study items rated on 8 point scale. The response scores for each item in terms of probability are presented in column-3. The actual response score in terms of Likert’s scale for each study item is presented in colum-4. The overall score is computed with the assumptions of the study items are equally weighted as per column-5 and with changing weights as per column-7. The probability of selecting each study item with equal weight is presented in column-6. In this example, it is assumed that the first item in the list is the most weighted, and so on the last item in the list has least weight. Hence the weights are allocated accordingly and presented in column-7. The probabilities of changing weights are presented in column-8. Model-1 and model-2 are constructed based on columns 4, 6 and 8 where as Model-3 and Model-4 are constructed with the columns 2, 6 & 8 and the rows 10, 11&12. Model-1 computed the study score based on items in the list where as Model-3 also computed the same study score, but it is based on the score point of the scale. Hence Model-1 and Model-3 are same to compute the study scores by considering the assumption of listed items are equally weighted to influence the score. Model-2 computed the study score based on item in the list where as Model-4 also computed the same score, but it is based on the score point of the scale. Hence Model-2 and Model-4 are same to compute the study scores by considering the assumption of listed items are not equally weighted to influence the score.

Table-3.2 has presented the results of various descriptive statistics based on the calculations of moments. MS Excel template is developed for calculating the values. It has facilitated to calculate the changing scenario of study scorings. This template is more flexible in exploring dynamic options of responses. As per the presented illustration, the average study score with equal weighted list of items is 4.4211 whereas the same score with varying weights is 4.5158. It is observed that the expected score in with weighted list is more than un-weighted list. The variance of the score with un-weighted list is 23.3684 whereas the same with weighted list is 24.3684. The coefficient of variation with un-weighted list is 0.4422 whereas the same with weighted list is 0.4416. Hence, it is observed that weighted list study score is more consistent than the un-weighted list of items. Both the cases exhibit the negative skewness with the coefficients 0.1349 with equal weighted list and 0.0950 with unequal weighted list. Hence it is observed that the weighted list has less skewness when compared to un-weighted list. More relevant inferences can be done with the obtained results. This study can be extended to more tools of scaling for the comparative analysis.

Table 3.3: Joint Probability Distribution of Score Response with Equal Weighted Items for Models 2.2.1 & 2.2.2

		Scale (j)										
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Total**	**wt-1**	**k**
Study Item (i)	1	0	0	0	0	0.053	0	0	0	0.053	1	5
	2	0	0	0	0.053	0	0	0	0	0.053	1	4
	3	0	0	0	0	0	0.053	0	0	0.053	1	6
	4	0	0	0	0.053	0	0	0	0	0.053	1	4
	5	0	0	0	0	0	0	0	0.053	0.053	1	8
	6	0	0.053	0	0	0	0	0	0	0.053	1	2
	7	0.053	0	0	0	0	0	0	0	0.053	1	1
	8	0	0	0	0	0	0	0.053	0	0.053	1	7
	9	0	0	0	0.053	0	0	0	0	0.053	1	4
	10	0	0	0	0	0	0.053	0	0	0.053	1	6
	11	0	0	0.053	0	0	0	0	0	0.053	1	3
	12	0	0	0	0	0.053	0	0	0	0.053	1	5
	13	0	0.053	0	0	0	0	0	0	0.053	1	2
	14	0	0	0	0	0	0	0.053	0	0.053	1	7
	15	0.053	0	0	0	0	0	0	0	0.053	1	1
	16	0	0	0	0.053	0	0	0	0	0.053	1	4
	17	0	0	0	0	0	0.053	0	0	0.053	1	6
	18	0	0	0	0	0.053	0	0	0	0.053	1	5
	19	0	0	0	0.053	0	0	0	0	0.053	1	4
	QOJ(1)	0.105	0.105	0.053	0.263	0.158	0.158	0.105	0.053	1	19	84
	J	1	2	3	4	5	6	7	8			

While finding the joint probability distribution, the relation of all the selected options have equal chances of they are being involved in scoring process. This table is according to the previous spontaneous selection criteria of the responses. As the number of listed items are 19, the probability for each allocated cell is 0.053, 'k' is score for each item by the respondent. The value of 'k' is in between 1 to 8 in this example.

Table 3.4: Joint Probability Distribution of Score Response with Unequal Weighted Items for Models 2.2.3 & 2.2.4

		Score Point Scale (J)										
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**qi0**	**wt-2**	**k**
Study Items (I)	1	0	0	0	0	0.1	0	0	0	0.1	19	5
	2	0	0	0	0.0947	0	0	0	0	0.0947	18	4
	3	0	0	0	0	0	0.0895	0	0	0.0895	17	6
	4	0	0	0	0.0842	0	0	0	0	0.0842	16	4
	5	0	0	0	0	0	0	0	0.0789	0.0789	15	8
	6	0	0.0737	0	0	0	0	0	0	0.0737	14	2
	7	0.0684	0	0	0	0	0	0	0	0.0684	13	1
	8	0	0	0	0	0	0	0.0632	0	0.0632	12	7
	9	0	0	0	0.0579	0	0	0	0	0.0579	11	4
	10	0	0	0	0	0	0.0526	0	0	0.0526	10	6
	11	0	0	0.0474	0	0	0	0	0	0.0474	9	3
	12	0	0	0	0	0.0421	0	0	0	0.0421	8	5
	13	0	0.0368	0	0	0	0	0	0	0.0368	7	2
	14	0	0	0	0	0	0	0.0316	0	0.0316	6	7
	15	0.0263	0	0	0	0	0	0	0	0.0263	5	1
	16	0	0	0	0.0211	0	0	0	0	0.0211	4	4
	17	0	0	0	0	0	0.0158	0	0	0.0158	3	6
	18	0	0	0	0	0.0105	0	0	0	0.0105	2	5
	19	0	0	0	0.0053	0	0	0	0	0.0053	1	4
	q0j	0.0947	0.1105	0.0474	0.2632	0.1526	0.1579	0.0947	0.0789	1	190	84
	J	1	2	3	4	5	6	7	8			

While finding the above joint probability distribution, the relation of all the selected options have unequal chances of they are being involved in scoring process. This table is according to the previous spontaneous selection criteria of the responses. As the number of listed items are 19, and the order of their weights are from 19 to 1, the probability for each allocated cell is accordingly. k is score for each item by the respondent. The value of 'k' is in between 1 to 8 in this example.

Table 3.2: Various Descriptive Statistics with Fixed and Varying Weights of Study Items for Models 2.2.1. to 2.2.4

Statistic	Model-2.2.1	Model-2.2.2	Model-2.2.3	Model- 2.2.4
Myu1'(I)	4.4211	4.5158	4.4211	4.5158
Myu2'(I)	23.3684	24.3684	23.3684	24.3684
Myu3'(I)	136.1053	145.2000	136.1053	145.2000
Myu4'(I)	845.0526	923.9263	845.0526	923.9263
Mean (I)	4.4211	4.5158	4.4211	4.5158
Myu2(I)	3.8227	3.9761	3.8227	3.9761
SD(I)	1.9552	1.9940	1.9552	1.9940
CV(I)	0.4422	0.4416	0.4422	0.4416
Myu3(I)	-1.0086	-0.7528	-1.0086	-0.7528
Beta-1(I)	0.0182	0.0090	0.0182	0.0090
Gamma-1(I)	0.1349	0.0950	0.1349	0.0950
Myu4(I)	32.5484	35.1883	32.5484	35.1883
Beta2(I)	2.2273	2.2258	2.2273	2.2258
Gamma2(I)	-0.7727	-0.7742	-0.7727	-0.7742

From the above numerical illustrations it is observed that the statistical values are equal for the models 2.1.1, 2.1.3, 2.2.1 and 2.2.3 for equal weight listed items. Whereas another set of models provide the unequal weighted items are, 2.1.2, 2.1.4, 2.2.2. and 2.2.4. provide the same values.

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